

Optimization of Compressive Strength of Concrete Made with Partial Replacement of Cement with Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) using Scheffe's (6,3) Model

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ABSTRACT

This research work is aimed at using Scheffe's Third Degree Model for six component mixtures, Scheffe's (6,3) to optimize the compressive strength of concrete made through partial replacement of 60 percent of cement with 30 percent of Cassava Peel Ash (CPA) and 30 percent of Rice Husk Ash (RHA). Before now, Nwachukwu and others (2022i) has carried out research on the subject matter based on the Scheffe's second degree model. Due to the upper hand that the third degree has over the second degree in terms of improved compressive strength, this recent work has become very essential. Through the use of Scheffe's Simplex optimization method, the compressive strengths of the present work based on the third degree model were obtained for 112 different mix proportions. Control experiments were also carried out, and the compressive strengths evaluated. The adequacy of the third degree model was confirmed through the use of the Student's t-test statistics. The highest compressive strength was obtained as 43.75 MPa which is slightly higher than the maximum value obtained by Nwachukwu and others (2022i) based on the second degree model. Again, the maximum value is higher than the minimum value specified by the American Concrete Institute (ACI), as 20 MPa and also the minimum value specified by ASTM C 39 or ASTM C 469, as 30.75 for good concrete. Thus, the compressive strength value can sustain construction of light-weight structures such as construction of Walkways, Pavement slabs etc and some heavy-weight structures such as Bridges, Airports etc at the best possible economic and safety advantages.

KEYWORDS: Concrete, CPA, RHA, Cement, Scheffe's (6,3) Optimization/Polynomial Model, Compressive Strength, Mixture Design

1. INTRODUCTION

From experience in the construction industries, Cement Cost Factor (CCF) constitutes almost 50 percent of the Overall Building Cost Factor (OBCF). Therefore in a bit to solve the world's rising cost of building materials especially cement, there have been urgent needs to partially replace cement with some locally and environmentally friendly made binders. Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) as agricultural waste products fall into this category. They are inexpensive binders and have the ability to reduce

the cement requirement or decrease the overall production cost of concrete. Subsequently, reduction in cement requirement leads to less environmental pollution by cement factories as well as promoting economic and environmental benefits of disposing agricultural waste product. Overall, CPA and RHA provide good compressive strength to the concrete.

Concrete as a homogeneous mixture of cement, sand, gravel and water is very strong in carrying

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compressive forces and as result, is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). It is the most widely used construction material in the universe. As stated by Oyenuga (2008), concrete is a composite inert material comprising of a binder course (cement), mineral filler or aggregates and water. Again, concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. According to Shetty (2006), concrete, especially plain type possesses a very low tensile strength, limited ductility and little resistance to cracking. This has resulted to continuous search for upgrading the properties of concrete, especially the cement contribution. Many researches have shown that CPA and RHA as local inexpensive and environmentally friendly binders can partially replace cement with utmost positive results in terms of high quality concrete production. The use of these two binders, CPA and RHA, somehow can improve the efficiency of the cement and henceforth the concrete mixture due to the outstanding qualities and inherent properties both possess. For instance, RHA contains over 85% of amorphous silica by weight. Then, as a pozzolanic reactive material, it can be used to improve surface area of transition zone between the microscopic structure of cement paste and aggregate in the high performance concrete. For the CPA, because it is an agricultural solid waste derivative, its utilization as a supplementary cementitious material in the production of concrete is vital and necessary because it supports the reuse and recycling of solid wastes in line with environmental sustainability. As in the case of the previous work of Nwachukwu and others (2022i), greater efficiency for the mixture design of concrete made with cement that is partially replaced with CPA and RHA can be carried out through optimization. By all standards, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. Specifically, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as workability, strength and durability.

The design of concrete mix according to (Shetty, 2006) is not a simple task on the account of the widely varying properties of the constituent

materials, as well as the conditions that prevail at the site of work, the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed. Again, concrete mix design according to Jackson and Dhir (1996) is the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. Thus, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the empirical methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) seems complex and time consuming as they involve a lot of trial mixes and deep statistical calculations before the desired strength of the concrete can be achieved. According to Shacklock (1974), the objective of mix design has been to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work. Thus, optimization of the concrete mixture design remains the fastest method, best option and the most efficient way of selecting concrete mix for better efficiency and performance of concrete when compared with usual empirical methods as listed above. A typical example of optimization model is Scheffe's Model, which can be in form of Scheffe's Second Degree model or Scheffe's Third Degree model. In this present work, Scheffe's Third Degree Model for six components mixtures (namely Water, Cement, CPA, RHA, Fine Aggregate and Coarse Aggregate) will be in focus.

The concrete's compressive strength is one of the most important properties of concrete that require close investigation because of its important role. Compressive strength of concrete is the strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine (UTM). Again, the compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete under examination.

This present work therefore examines the use of Scheffe's Third Degree Model in the optimization of the compressive strength of Concrete made with partial replacement of cement with CPA and RHA. Though work has been done on the present topic based on Scheffe's second degree model by Nwachukwu and others (2022i), but there are

proofs that results based on Scheffe's third degree always have edges over second degree model results. Also as expected, many related works have been done but none has been able to address the core subject matter wholly. For example, on CPA and RHA, Raheem and others (2015) carried out investigation on the effect of cassava peel ash (CPA) as alternative binder in concrete. Olatokunbo and others (2018) provided an assessment of strength properties of cassava peel ash-concrete. Mohd-Ashruddin and others (2017) assessed the chemical and morphological studies of cassava peel. Similarly, Ogbonna and others (2020) carried out an investigation into the characteristics and use of cassava peel ash in concrete production. Adetoye and others (2022), in their own contribution, investigated the compressive strength properties of cassava peel ash and wood ash in concrete production. Zareei and others (2017) investigated the role of Rice Husk Ash as a partial replacement of cement in high strength concrete containing micro silica. Again, Obute and others (2021) carried out the effect of the partial replacement of cement with cassava peel ash and rice husk ash on concrete.

In the strategic area of optimization, recent works have shown that many researchers have used Scheffe's methods to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezech and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezech and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezech and others (2010b) optimized the aggregate composition of laterite/sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe's mathematical model in the optimization of compressive strength of Periwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete

(GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. Nwachukwu and others (2022h) also used the Scheffe's second degree model to optimize the compressive strength of HPSFRC. Nwachukwu and others (2022i) applied the Scheffe's second degree model in the optimization of compressive strength of concrete made with partial replacement of cement with Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC). Finally, Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). From the foregoing, it appears that the subject matter has not been wholly addressed as it can be envisaged that no work has been done on the use of Scheffe's Third Degree Model to optimize the compressive strength of concrete made with partial replacement of cement with Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Henceforth, the need for this present research work.

2. SCHEFFE'S (6, 3) MODEL BASICS

In general, a simplex lattice is defined as a structural representation of lines joining the atoms of a mixture. However, these atoms are constituent components of the mixture. For the present concrete mixture in the Scheffe's third degree model, the six constituent elements are, Water, Cement, CPA,

RHA, Fine Aggregate, and Coarse Aggregate. According to Obam (2009), mixture components are usually subject to the restriction that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1; \Rightarrow \sum_{i=1}^q X_i = 1 \quad (1)$$

where $X_i \geq 0$ and $i = 1, 2, 3, \dots, q$, and q = the number of mixtures.

2.1. SIX COMPONENT MIXTURES IN SCHEFFE'S SIMPLEX LATTICE DESIGN

The (q, m) , for example $(6,3)$ simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen mathematical equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes $(m+1)$ equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degree, which in this present study is 2.

For example a $(3, 2)$ lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^4C_2 = 6$ points. Each X_i can take $m+1 = 3$ possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{2}, \frac{1}{2}, 0), (0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2})$. The general formula for evaluating the number of coefficients/terms/points required for a given lattice is always given by:

$$k = \frac{(q+m-1)!}{(q-1)!m!} \quad \text{Or} \quad {}^{q+m-1}C_m \quad 2(a-b)$$

Where k = number of coefficients/ terms / points, q = number of components = 6 in this study and m = number of degree of polynomial = 3 in this present work

Using either of Eqn. (2), $k_{(6,3)} = 56$

Below are the possible design points for Scheffe's $(6,3)$ lattice.

$A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0); A_5 (0,0,0,0,1,0); A_6 (0,0,0,0,0,1); A_{112} (0.67,0.33,0,0,0,0); A_{122} (0.67,0,0.33,0,0,0); A_{113} (0.67,0,0,0.33,0,0); A_{133} (0.67,0,0,0,0.33,0); A_{114} (0.67,0,0,0,0,0.33); A_{144} (0,0.67,0.33,0,0,0); A_{115} (0,0.67,0,0.33,0,0); A_{155} (0,0.67,0,0,0.33,0); A_{116} (0,0.67,0,0,0,0.33); A_{166} (0,0,0.67,0.33,0,0); A_{223} (0,0,0.67,0,0.33,0); A_{233} (0,0,0.67,0,0,0.33); A_{224} (0,0,0,0.67,0.33,0); A_{244} (0,0,0,0.67,0,0.33); A_{225} (0,0,0,0,0.67,0.33); A_{255} (0.50,0.50,0,0,0,0); A_{226} (0.50,0,0.50,0,0,0); A_{266} (0.50,0,0,0.50,0,0); A_{334} (0.50,0,0,0,0.50,0); A_{344} (0.50,0,0,0,0,0.50); A_{335} (0,0.50,0.50,0,0,0); A_{355} (0,0.50,0,0.50,0,0); A_{336} (0,0.50,0,0,0.50,0); A_{366} (0,0.50,0,0,0,0.50); A_{445} (0,0,0.50,0.50,0,0); A_{455} (0,0,0.50,0,0.50,0); A_{446} (0,0,0.50,0,0,0.50); A_{466} (0,0,0.50,0.50,0,0); A_{556} (0,0,0,0.50,0.50,0); A_{566} (0,0,0,0,0.50,0.50); A_{123} (0.80,0.20,0,0,0,0); A_{124} (0.80,0,0.20,0,0,0); A_{125} (0.80,0,0,0.20,0,0); A_{126} (0.80,0,0,0,0.20,0); A_{134} (0.80,0,0,0,0.20,0); A_{135} (0,0.80,0.20,0,0,0); A_{136} (0,0.80,0,0.20,0,0); A_{145} (0,0.80,0,0,0.20,0); A_{146} (0,0.80,0,0,0.20,0); A_{156} (0,0,0.80,0.20,0,0); A_{234} (0,0,0.80,0,0.20,0); A_{235} (0,0,0.80,0,0,0.20); A_{236} (0,0,0.80,0.20,0,0); A_{245} (0,0,0,0.80,0,0.20); A_{246} (0,0,0,0,0.80,0.20); A_{256} (0.60,0.40,0,0,0,0); A_{345} (0.60,0,0.40,0,0,0); A_{346} (0.60,0,0,0.40,0,0); A_{356} (0.60,0,0,0,0.40,0); A_{456} (0.60,0,0,0,0,0.40) \quad (3)$

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \dots X_q$ is given in the form of Eqn.(4)

$$N = b_0 + \sum b_i x_i + \sum b_{ij} x_j + \sum b_{ijk} x_k + \dots + \sum b_{i_1 i_2 \dots i_n} x_{i_1} x_{i_2} \dots x_{i_n} \quad (4)$$

where $(1 \leq i \leq q, 1 \leq i \leq j \leq k \leq q, 1 \leq i_1 \leq i_2 \leq \dots \leq i_n \leq q)$ respectively, b = constant coefficients and N is the response which represents the property under investigation, which, in this case is the compressive strength.

As this research work is based on the Scheffe's $(6, 3)$ simplex, the actual form of Eqn. (4) for six component mixture, degree three, $(6, 3)$ will be formulated in this present work.

2.2. ACTUAL and PSEUDO COMPONENTS.

The relationship between the pseudo components and the actual components in the Scheffe's mix design is given by:

$$Z = A * X \quad (5)$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields:

$$X = A^{-1} * Z \quad (6)$$

2.3. FORMULATION of REGRESSION EQUATION FOR SCHEFFE'S (6, 3) LATTICE FOR THE PRESENT CONCRETE MIXTURE

The polynomial equation by Scheffe (1958), which is known as response is given in Eqn.(4). and for the Scheffe's (6,3) simplex lattice, the polynomial equation for six component mixtures in second degree capacity has been formulated based on Eqn.(4) by the work of Nwachukwu and others (2022g). The expansion of this work based on third degree capacity is as shown under:

$$\begin{aligned} N = & \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 \\ & + \beta_{16} X_1 X_6 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{26} X_2 X_6 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{36} X_3 X_6 \\ & + \beta_{45} X_4 X_5 + \beta_{46} X_4 X_6 + \beta_{56} X_5 X_6 + \beta_{123} X_1 X_2 X_3 + \beta_{124} X_1 X_2 X_4 + \beta_{125} X_1 X_2 X_5 + \beta_{126} X_1 X_2 X_6 \\ & + \beta_{134} X_1 X_3 X_4 + \beta_{135} X_1 X_3 X_5 + \beta_{136} X_1 X_3 X_6 + \beta_{145} X_1 X_4 X_5 + \beta_{146} X_1 X_4 X_6 + \beta_{234} X_2 X_3 X_4 \\ & + \beta_{235} X_2 X_3 X_5 + \beta_{236} X_2 X_3 X_6 + \beta_{245} X_2 X_4 X_5 + \beta_{246} X_2 X_4 X_6 + \beta_{256} X_2 X_5 X_6 + \beta_{345} X_3 X_4 X_5 \\ & + \beta_{346} X_3 X_4 X_6 + \beta_{356} X_3 X_5 X_6 + \beta_{456} X_4 X_5 X_6 + \gamma_{12} X_1 X_2^2 + \gamma_{13} X_1 X_3^2 + \gamma_{14} X_1 X_4^2 + \gamma_{15} X_1 X_5^2 + \gamma_{16} X_1 X_6^2 \\ & + \gamma_{23} X_2 X_3^2 + \gamma_{24} X_2 X_4^2 + \gamma_{25} X_2 X_5^2 + \gamma_{26} X_2 X_6^2 + \gamma_{34} X_3 X_4^2 + \gamma_{35} X_3 X_5^2 + \gamma_{36} X_3 X_6^2 + \gamma_{45} X_4 X_5^2 + \gamma_{46} X_4 X_6^2 \\ & + \gamma_{56} X_5 X_6^2 \end{aligned} \quad (7)$$

2.4. SCHEFFE'S (6, 3) MODEL COEFFICIENTS EQUATIONS FOR THE PRESENT CONCRETE MIXTURE

Let N_i = Response Function for the Pure Component, i

Then, through the expansion of the work of Obam (2006), it can be established that:

$$\beta_1 = N_1; \beta_2 = N_2; \beta_3 = N_3; \beta_4 = N_4; \beta_5 = N_5 \text{ and } \beta_6 = N_6 \quad \mathbf{8(a-f)}$$

$$\beta_{12} = 2.25(N_{112} + N_{122} - N_1 - N_2); \beta_{13} = 2.25(N_{113} + N_{133} - N_1 - N_3); \beta_{14} = 2.25(N_{114} + N_{144} - N_1 - N_4); \quad \mathbf{9(a-c)}$$

$$\beta_{15} = 2.25(N_{115} + N_{155} - N_1 - N_5); \beta_{16} = 2.25(N_{116} + N_{166} - N_1 - N_6); \beta_{23} = 2.25(N_{223} + N_{233} - N_2 - N_3); \quad \mathbf{10(a-c)}$$

$$\beta_{24} = 2.25(N_{224} + N_{244} - N_2 - N_4); \beta_{25} = 2.25(N_{225} + N_{255} - N_2 - N_5); \beta_{26} = 2.25(N_{226} + N_{266} - N_2 - N_6); \quad \mathbf{11(a-c)}$$

$$\beta_{34} = 2.25(N_{334} + N_{344} - N_3 - N_4); \beta_{35} = 2.25(N_{335} + N_{355} - N_3 - N_5); \beta_{36} = 2.25(N_{336} + N_{366} - N_3 - N_6); \quad \mathbf{12(a-c)}$$

$$\beta_{45} = 2.25(N_{445} + N_{455} - N_4 - N_5); \beta_{46} = 2.25(N_{446} + N_{466} - N_4 - N_6); \beta_{56} = 2.25(N_{556} + N_{566} - N_5 - N_6); \quad \mathbf{13(a-c)}$$

$$\beta_{123} = 27N_{123} - 6.75(N_{112} + N_{122} + N_{113} + N_{133} + N_{223} + N_{233}) + 2.25(N_1 + N_2 + N_3) \quad \mathbf{(14)}$$

$$\beta_{124} = 27N_{124} - 6.75(N_{112} + N_{122} + N_{114} + N_{144} + N_{224} + N_{244}) + 2.25(N_1 + N_2 + N_4) \quad \mathbf{(15)}$$

$$\beta_{125} = 27N_{125} - 6.75(N_{112} + N_{122} + N_{115} + N_{155} + N_{225} + N_{255}) + 2.25(N_1 + N_2 + N_5) \quad \mathbf{(16)}$$

$$\beta_{126} = 27N_{126} - 6.75(N_{112} + N_{122} + N_{116} + N_{166} + N_{226} + N_{266}) + 2.25(N_1 + N_2 + N_6) \quad \mathbf{(17)}$$

$$\beta_{134} = 27N_{134} - 6.75(N_{113} + N_{133} + N_{114} + N_{144} + N_{334} + N_{344}) + 2.25(N_1 + N_3 + N_4) \quad \mathbf{(18)}$$

$$\beta_{135} = 27N_{135} - 6.75(N_{113} + N_{133} + N_{115} + N_{155} + N_{335} + N_{355}) + 2.25(N_1 + N_3 + N_5) \quad \mathbf{(19)}$$

$$\beta_{136} = 27N_{136} - 6.75(N_{113} + N_{133} + N_{116} + N_{166} + N_{336} + N_{366}) + 2.25(N_1 + N_3 + N_6) \quad \mathbf{(20)}$$

$$\beta_{145} = 27N_{145} - 6.75(N_{114} + N_{144} + N_{115} + N_{155} + N_{445} + N_{455}) + 2.25(N_1 + N_4 + N_5) \quad \mathbf{(21)}$$

$$\beta_{146} = 27N_{146} - 6.75(N_{114} + N_{144} + N_{116} + N_{166} + N_{446} + N_{466}) + 2.25(N_1 + N_4 + N_6) \quad \mathbf{(22)}$$

$$\beta_{156} = 27N_{156} - 6.75(N_{115} + N_{155} + N_{116} + N_{166} + N_{556} + N_{566}) + 2.25(N_1 + N_5 + N_6) \quad \mathbf{(23)}$$

$$\beta_{234} = 27N_{234} - 6.75(N_{223} + N_{233} + N_{224} + N_{244} + N_{334} + N_{344}) + 2.25(N_2 + N_3 + N_4) \quad \mathbf{(24)}$$

$$\beta_{235} = 27N_{235} - 6.75(N_{223} + N_{233} + N_{225} + N_{255} + N_{335} + N_{355}) + 2.25(N_2 + N_3 + N_5) \quad \mathbf{(25)}$$

$$\beta_{236} = 27N_{236} - 6.75(N_{223} + N_{233} + N_{226} + N_{266} + N_{336} + N_{366}) + 2.25(N_2 + N_3 + N_6) \quad \mathbf{(26)}$$

$$\beta_{245} = 27N_{245} - 6.75(N_{224} + N_{244} + N_{225} + N_{255} + N_{445} + N_{455}) + 2.25(N_2 + N_4 + N_5) \quad \mathbf{(27)}$$

$$\beta_{246} = 27N_{246} - 6.75(N_{224} + N_{244} + N_{226} + N_{266} + N_{446} + N_{466}) + 2.25(N_2 + N_4 + N_6) \quad \mathbf{(28)}$$

$$\beta_{256} = 27N_{256} - 6.75(N_{225} + N_{255} + N_{226} + N_{266} + N_{556} + N_{566}) + 2.25(N_2 + N_5 + N_6) \quad \mathbf{(29)}$$

$$\beta_{345} = 27N_{345} - 6.75(N_{334} + N_{344} + N_{335} + N_{355} + N_{445} + N_{455}) + 2.25(N_3 + N_4 + N_5) \quad \mathbf{(30)}$$

$$\beta_{346} = 27N_{346} - 6.75(N_{334} + N_{344} + N_{336} + N_{366} + N_{446} + N_{466}) + 2.25(N_3 + N_4 + N_6) \quad \mathbf{(31)}$$

$$\beta_{356} = 27N_{356} - 6.75(N_{335} + N_{355} + N_{336} + N_{366} + N_{556} + N_{566}) + 2.25(N_3 + N_5 + N_6) \quad \mathbf{(32)}$$

$$\beta_{456} = 27N_{456} - 6.75(N_{445} + N_{455} + N_{446} + N_{466} + N_{556} + N_{566}) + 2.25(N_4 + N_5 + N_6) \quad \mathbf{(33)}$$

$$\gamma_{12} = 2.25(3N_{112}+3N_{122}-N_1+N_2); \gamma_{13} = 2.25(3N_{113}+3N_{133}-N_1+N_3); \gamma_{14} = 2.25(3N_{114}+3N_{144}-N_1+N_4); \mathbf{34(a-c)}$$

$$\gamma_{15} = 2.25(3N_{115}+3N_{155}-N_1+N_5); \gamma_{16} = 2.25(3N_{116}+3N_{166}-N_1+N_6); \gamma_{23} = 2.25(3N_{223}+3N_{233}-N_2+N_3); \mathbf{35(a-c)}$$

$$\gamma_{24} = 2.25(3N_{224}+3N_{244}-N_2+N_4); \gamma_{25} = 2.25(3N_{225}+3N_{255}-N_2+N_5); \gamma_{26} = 2.25(3N_{226}+3N_{266}-N_2+N_6); \mathbf{36(a-c)}$$

$$\gamma_{34} = 2.25(3N_{334}+3N_{344}-N_3+N_4); \gamma_{35} = 2.25(3N_{335}+3N_{355}-N_3+N_5); \gamma_{36} = 2.25(3N_{336}+3N_{366}-N_3+N_6); \mathbf{37(a-c)}$$

$$\gamma_{45} = 2.25(3N_{445}+3N_{455}-N_4+N_5); \gamma_{46} = 2.25(3N_{446}+3N_{466}-N_4+N_6); \gamma_{56} = 2.25(3N_{556}+3N_{566}-N_4+N_6) \mathbf{38(a-c)}$$

2.5. SCHEFFE'S (6, 3) MIXTURE DESIGN MODEL FOR THE PRESENT CONCRETE MIXTURE

When we substitute Eqns. (8)-(38) into Eqn. (7), we obtain the mixture design model for the present concrete mixture based on Scheffe's (6,3) lattice.

2.6. ACTUAL and PSEUDO MIX PROPORTIONS FOR THE PRESENT CONCRETE MIXTURE BASED ON SCHEFFE'S (6,3) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINT and CONTROL TEST POINT

2.6.1. AT THE INITIAL EXPERIMENTAL TEST POINTS

Based on Eqn. (1), the requirement of simplex lattice design criteria makes it impossible to use the conventional mix ratios such as 1:2:4 etc., at a given water/cement ratio for the actual mix ratio. Thus, there is need for the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix ratios are always chosen for the six vertices of Scheffe's (6,3) lattice as under. But note that cement is partially replaced with CPA and RHA in the ratio of C:CPA: RHA = 0.4: 0.3:0.3.

$$A_1 (0.67:0.4:0.3:0.3:1.7:2.0); A_2 (0.56:0.4:0.3:0.3:1.6:1.8); A_3 (0.5:0.4:0.3:0.3:1.2:1.7); A_4(0.7:0.4:0.3:0.3:1:1.8); A_5(0.75:0.4:0.3:0.3:1.3:1.2); \text{and } A_6(0.80:0.4:0.3:0.3:1.3:1.2); \mathbf{(39)}$$

Which represent Water/Cement Ratio, Cement, CPA, RHA, Fine Aggregate and Coarse Aggregate.

However, a factor of 0.4 can be used to divide through Eqn.(39), to make the quantity of cement to be unity since the measurement of other components are dependent on cement. Thus Eqn. (39) can be rewritten as:

$$A_1 (1.7:1.0:0.8:0.8:4.3:5.0); A_2 (1.4:1.0:0.8:0.8:4.0:4.5); A_3 (1.3:1.0:0.8:0.8:3.0:4.3); A_4(1.8:1.0:0.8:0.8:2.5:6.3); A_5(1.9:1.0:0.8:0.8:3.3:3.0); \text{and } A_6(2.0:1.0:0.8:0.8:3.3:3.0) \mathbf{(40)}$$

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures are always chosen:

$$A_1(1:0:0:0:0:0), A_2(0:1:0:0:0:0), A_3(0:0:1:0:0:0), A_4(0:0:0:1:0:0), A_5(0:0:0:0:1:0) \text{ and } A_6(0:0:0:0:0:1) \mathbf{(41)}$$

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. Substituting the mix ratios from point A_1 in Eqn.(41) into Eqn. (5) yields:

$$\begin{pmatrix} 1.70 \\ 1 \\ 0.8 \\ 0.8 \\ 4.3 \\ 5.0 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} & A_{116} \\ A_{221} & A_{222} & A_{223} & A_{224} & A_{225} & A_{226} \\ A_{331} & A_{332} & A_{333} & A_{334} & A_{335} & A_{336} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} & A_{446} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} & A_{556} \\ A_{661} & A_{662} & A_{663} & A_{664} & A_{665} & A_{666} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \mathbf{(42)}$$

Transforming the R.H.S matrix and solving, we obtain

$$A_{111}= 1.70; A_{221}= 1; A_{331}= 0.8; A_{441}= 0.8; A_{551}= 4.3; A_{661}= 5.0$$

The same approach is used to obtain the remaining values as shown in Eqn. (43)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 1.7 & 1.4 & 1.3 & 1.8 & 1.9 & 2.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 4.3 & 4.0 & 3.0 & 2.5 & 3.3 & 3.3 \\ 5.0 & 4.5 & 4.3 & 6.3 & 3.0 & 3.0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} \mathbf{(43)}$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(43) will yield the corresponding actual mix ratios.

For instance, considering point A₁₁₂ we have: A₁₂ (0.67,0.33, 0, 0, 0, 0), and the following equation results:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 1.7 & 1.4 & 1.3 & 1.8 & 1.9 & 2.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 4.3 & 4.0 & 3.0 & 2.5 & 3.3 & 3.3 \\ 5.0 & 4.5 & 4.3 & 6.3 & 3.0 & 3.0 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.6 \\ 1.0 \\ 0.8 \\ 0.8 \\ 3.9 \\ 4.8 \end{pmatrix} \quad (44)$$

Solving, Z₁ = 1.6; Z₂ = 1.0; Z₃ = 0.8; Z₄ = 0.8; Z₅ = 3.9 and Z₆ = 4.8

The same approach goes for the remaining mid-point mix ratios.

Thus, fifty-six (56) experimental tests will be carried out in order to generate the polynomial coefficients and the corresponding mix ratios are depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio For The present CPA-RHA Cement Concrete Based on Scheffe's (6,3) Lattice.

S/N	POINTS	PSEUDO COMPONENT						RESPONSE SYMBOL	ACTUAL COMPONENT					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆
1.	1	1	0	0	0	0	0	N ₁	1.7	1.0	0.8	0.8	0.5	0.5
2.	2	0	1	0	0	0	0	N ₂	1.4	1.0	0.8	0.8	4.0	4.5
3.	3	0	0	1	0	0	0	N ₃	1.3	1.0	0.8	0.8	3.0	4.3
4.	4	0	0	0	1	0	0	N ₄	1.8	1.0	0.8	0.8	2.5	6.3
5.	5	0	0	0	0	1	0	N ₅	1.9	1.0	0.8	0.8	3.0	3.3
6.	6	0	0	0	0	0	1	N ₆	2.0	1.0	0.8	0.8	3.0	3.3
7.	112	0.67	0.33	0	0	0	0	N ₁₁₂	1.6	1.0	0.8	0.8	3.9	4.8
8.	122	0.67	0	0.33	0	0	0	N ₁₂₂	1.6	1.0	0.8	0.8	3.0	4.2
9.	113	0.67	0	0	0.33	0	0	N ₁₁₃	1.8	1.0	0.8	0.8	2.5	3.5
10.	133	0.67	0	0	0	0.33	0	N ₁₃₃	1.9	1.0	0.8	0.8	3.3	4.7
11.	114	0.67	0	0	0	0	0.33	N ₁₁₄	1.7	1.0	0.8	0.8	3.3	4.5
12.	144	0	0.67	0.33	0	0	0	N ₁₄₄	2.0	1.0	0.8	0.8	3.2	4.8
13.	115	0	0.67	0	0.33	0	0	N ₁₁₅	1.8	1.0	0.8	0.8	3.4	5.0
14.	155	0	0.67	0	0	0.33	0	N ₁₅₅	1.6	1.0	0.8	0.8	3.0	4.5
15.	116	0	0.67	0	0	0	0.33	N ₁₁₆	1.8	1.0	0.8	0.8	3.0	4.8
16.	166	0	0	0.67	0.33	0	0	N ₁₆₆	1.9	1.0	0.8	0.8	3.4	4.7
17.	223	0	0	0.67	0	0.33	0	N ₂₂₃	1.8	1.0	0.8	0.8	2.5	4.8
18.	233	0	0	0.67	0	0	0.33	N ₂₃₃	1.6	1.0	0.8	0.8	3.0	4.6
19.	224	0	0	0	0.67	0.33	0	N ₂₂₄	1.9	1.0	0.8	0.8	2.5	5.0
20.	244	0	0	0	0.67	0	0.33	N ₂₄₄	2.0	1.0	0.8	0.8	2.8	4.8
21.	225	0	0	0	0	0.67	0.33	N ₂₂₅	1.8	1.0	0.8	0.8	3.0	4.8
22.	255	0.50	0.50	0	0	0	0	N ₂₅₅	1.9	1.0	0.8	0.8	0.5	0.5
23.	226	0.50	0	0.50	0	0	0	N ₂₂₆	1.5	1.0	0.8	0.8	4.0	4.5
24.	266	0.50	0	0	0.50	0	0	N ₂₆₆	1.4	1.0	0.8	0.8	3.0	4.3
25.	334	0.50	0	0	0	0.50	0	N ₃₃₄	1.9	1.0	0.8	0.8	2.5	6.3
26.	344	0.50	0	0	0	0	0.50	N ₃₄₄	1.8	1.0	0.8	0.8	3.0	3.3
27.	335	0	0.50	0.50	0	0	0	N ₃₃₅	2.1	1.0	0.8	0.8	3.0	3.3
28.	355	0	0.50	0	0.50	0	0	N ₃₅₅	1.8	1.0	0.8	0.8	3.9	4.8
29.	336	0	0.50	0	0	0.50	0	N ₃₃₆	1.8	1.0	0.8	0.8	3.0	4.2
30.	366	0	0.50	0	0	0	0.50	N ₃₆₆	1.9	1.0	0.8	0.8	2.5	3.5
31.	445	0	0	0.50	0.50	0	0	N ₄₄₅	1.7	1.0	0.8	0.8	3.3	4.7
32.	455	0	0	0.50	0	0.50	0	N ₄₅₅	1.9	1.0	0.8	0.8	3.3	4.5
33.	446	0	0	0.50	0	0	0.50	N ₄₄₆	2.2	1.0	0.8	0.8	3.2	4.8
34.	466	0	0	0	0.50	0.50	0	N ₄₆₆	1.9	1.0	0.8	0.8	3.4	5.0

35.	556	0	0	0	0.50	0	0.50	N ₅₅₆	1.7	1.0	0.8	0.8	3.0	4.5
36.	566	0	0	0	0	0.50	0.50	N ₅₆₆	1.9	1.0	0.8	0.8	3.0	4.8
37.	123	0.80	0.20	0	0	0	0	N ₁₂₃	1.7	1.0	0.8	0.8	3.4	4.7
38.	124	0.80	0	0.20	0	0	0	N ₁₂₄	1.9	1.0	0.8	0.8	2.5	4.8
39.	125	0.80	0	0	0.20	0	0	N ₁₂₅	1.6	1.0	0.8	0.8	3.0	4.6
40.	126	0.80	0	0	0	0.20	0	N ₁₂₆	1.8	1.0	0.8	0.8	2.5	5.0
41.	134	0.80	0	0	0	0	0.20	N ₁₃₄	2.3	1.0	0.8	0.8	2.8	4.8
42.	135	0	0.80	0.20	0	0	0	N ₁₃₅	1.9	1.0	0.8	0.8	3.0	4.8
43.	136	0	0.80	0	0.20	0	0	N ₁₃₆	1.8	1.0	0.8	0.8	0.5	5.0
44.	145	0	0.80	0	0	0.20	0	N ₁₄₅	1.6	1.0	0.8	0.8	4.0	4.5
45.	146	0	0.80	0	0	0	0.20	N ₁₄₆	1.5	1.0	0.8	0.8	3.0	4.3
46.	156	0	0	0.80	0.20	0	0	N ₁₅₆	1.9	1.0	0.8	0.8	2.5	6.3
47.	234	0	0	0.80	0	0.20	0	N ₂₃₄	1.8	1.0	0.8	0.8	3.0	3.3
48.	235	0	0	0.80	0	0	0.20	N ₂₃₅	2.2	1.0	0.8	0.8	3.0	3.3
49.	236	0	0	0	0.80	0.20	0	N ₂₃₆	1.7	1.0	0.8	0.8	3.9	4.8
50.	245	0	0	0	0.80	0	0.20	N ₂₄₅	1.8	1.0	0.8	0.8	3.0	4.2
51.	246	0	0	0	0	0.80	0.20	N ₂₄₆	1.9	1.0	0.8	0.8	2.5	3.5
52.	256	0.60	0.40	0	0	0	0	N ₂₅₆	1.7	1.0	0.8	0.8	3.3	4.7
53.	345	0.60	0	0.40	0	0	0	N ₃₄₅	1.6	1.0	0.8	0.8	3.3	4.5
54.	346	0.60	0	0	0.40	0	0	N ₃₄₆	2.2	1.0	0.8	0.8	3.2	4.8
55.	356	0.60	0	0	0	0.40	0	N ₃₅₆	1.9	1.0	0.8	0.8	3.4	5.0
56.	456	0.60	0	0	0	0	0.40	N ₄₅₆	1.6	1.0	0.8	0.8	3.0	4.5

2.6.2. AT THE CONTROL TEST POINTS

Fifty - six (56) different controls will be predicted and according to Scheffe's (1958), their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2: Actual and Pseudo Component For The Present CPA-RHA Cement Concrete Based on Scheffe (6,3) Lattice for Control Points

S/N	POINTS	PSEUDO COMPONENT						CONTROL POINTS	ACTUAL COMPONENT					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆
1.	1	0.25	0.25	0.25	0.25	0	0	C ₁	0.61	1	0.8	0.8	1.38	1.83
2.	2	0.25	0.25	0.25	0	0.25	0	C ₂	0.62	1	0.8	0.8	1.45	1.68
3.	3	0.25	0.25	0	0.25	0.25	0	C ₃	0.67	1	0.8	0.8	1.40	1.70
4.	4	0.25	0	0.25	0.25	0.25	0	C ₄	0.66	1	0.8	0.8	1.30	1.68
5.	5	0	0.25	0.25	0.25	0.25	0	C ₅	0.63	1	0.8	0.8	1.28	1.63
6.	6	0.20	0.20	0.20	0.20	0.20	0	C ₆	0.64	1	0.8	0.8	1.36	1.70
7.	112	0.30	0.30	0.30	0.10	0	0	C ₁₁₂	0.59	1	0.8	0.8	1.45	1.83
8.	122	0.30	0.30	0.30	0	0.10	0	C ₁₂₂	0.59	1	0.8	0.8	1.48	1.77
9.	113	0.30	0.30	0	0.30	0.10	0	C ₁₁₃	0.65	1	0.8	0.8	1.42	1.80
10.	133	0.30	0	0.30	0.30	0.10	0	C ₁₃₃	0.64	1	0.8	0.8	1.30	1.77
11.	114	0	0.30	0.30	0.30	0.10	0	C ₁₁₄	0.60	1	0.8	0.8	1.27	1.71
12.	144	0.10	0.30	0.30	0.30	0	0	C ₁₄₄	0.60	1	0.8	0.8	1.31	1.79
13.	115	0.30	0.10	0.30	0.30	0	0	C ₁₁₅	0.62	1	0.8	0.8	1.33	1.83
14.	155	0.30	0.10	0.30	0.30	0	0	C ₁₅₅	0.63	1	0.8	0.8	1.41	1.85
15.	116	0.10	0.20	0.30	0.40	0	0	C ₁₁₆	0.61	1	0.8	0.8	1.25	1.79
16.	166	0.30	0.20	0.10	0.40	0	0	C ₁₆₆	0.64	1	0.8	0.8	1.35	1.85
17.	223	0.20	0.20	0.10	0.10	0.40	0	C ₂₂₃	1.40	1	0.8	0.8	1.04	1.59
18.	233	0.30	0.10	0.30	0.20	0.10	0	C ₂₃₃	0.62	1	0.8	0.8	1.36	1.77
19.	224	0.25	0.25	0.15	0.15	0.20	0	C ₂₂₄	0.61	1	0.8	0.8	1.51	3.16
20.	244	0.30	0.30	0.20	0.10	0.10	0	C ₂₄₄	0.68	1	0.8	0.8	1.56	1.96
21.	225	0.10	0.30	0.30	0.30	0	0	C ₂₂₅	1.30	1	0.8	0.8	1.31	1.79
22.	255	0.25	0.25	0.25	0.25	0	0	C ₂₅₅	0.64	1	0.8	0.8	1.38	1.83
23.	226	0.25	0.25	0.25	0	0.25	0	C ₂₂₆	0.65	1	0.8	0.8	1.45	1.68

24.	266	0.25	0.25	0	0.25	0.25	0	C ₂₆₆	0.64	1	0.8	0.8	1.40	1.70
25.	334	0.25	0	0.25	0.25	0.25	0	C ₃₃₄	0.67	1	0.8	0.8	1.30	1.68
26.	344	0	0.25	0.25	0.25	0.25	0	C ₃₄₄	0.65	1	0.8	0.8	1.28	1.63
27.	335	0.20	0.20	0.20	0.20	0.20	0	C ₃₃₅	0.66	1	0.8	0.8	1.36	1.70
28.	355	0.30	0.30	0.30	0.10	0	0	C ₃₅₅	0.57	1	0.8	0.8	1.45	1.83
29.	336	0.30	0.30	0.30	0	0.10	0	C ₃₃₆	0.56	1	0.8	0.8	1.48	1.77
30.	366	0.30	0.30	0	0.30	0.10	0	C ₃₆₆	0.63	1	0.8	0.8	1.42	1.80
31.	445	0.30	0	0.30	0.30	0.10	0	C ₄₄₅	0.62	1	0.8	0.8	1.30	1.77
32.	455	0	0.30	0.30	0.30	0.10	0	C ₄₅₅	0.63	1	0.8	0.8	1.27	1.71
33.	446	0.10	0.30	0.30	0.30	0	0	C ₄₄₆	0.63	1	0.8	0.8	1.31	1.79
34.	466	0.30	0.10	0.30	0.30	0	0	C ₄₆₆	0.66	1	0.8	0.8	1.33	1.83
35.	556	0.30	0.10	0.30	0.30	0	0	C ₅₅₆	0.65	1	0.8	0.8	1.41	1.85
36.	566	0.10	0.20	0.30	0.40	0	0	C ₅₆₆	0.65	1	0.8	0.8	1.25	1.79
37.	123	0.30	0.20	0.10	0.40	0	0	C ₁₂₃	0.67	1	0.8	0.8	1.35	1.85
38.	124	0.20	0.20	0.10	0.10	0.40	0	C ₁₂₄	1.44	1	0.8	0.8	1.04	1.59
39.	125	0.30	0.10	0.30	0.20	0.10	0	C ₁₂₅	0.66	1	0.8	0.8	1.36	1.77
40.	126	0.25	0.25	0.15	0.15	0.20	0	C ₁₂₆	0.65	1	0.8	0.8	1.51	3.16
41.	134	0.30	0.30	0.20	0.10	0.10	0	C ₁₃₄	0.69	1	0.8	0.8	1.56	1.96
42.	135	0.10	0.30	0.30	0.30	0	0	C ₁₃₅	1.34	1	0.8	0.8	1.31	1.79
43.	136	0.25	0.25	0.25	0.25	0	0	C ₁₃₆	0.64	1	0.8	0.8	1.38	1.83
44.	145	0.25	0.25	0.25	0	0.25	0	C ₁₄₅	0.66	1	0.8	0.8	1.45	1.68
45.	146	0.25	0.25	0	0.25	0.25	0	C ₁₄₆	0.69	1	0.8	0.8	1.40	1.70
46.	156	0.25	0	0.25	0.25	0.25	0	C ₁₅₆	0.63	1	0.8	0.8	1.30	1.68
47.	234	0	0.25	0.25	0.25	0.25	0	C ₂₃₄	0.65	1	0.8	0.8	1.28	1.63
48.	235	0.20	0.20	0.20	0.20	0.20	0	C ₂₃₅	0.67	1	0.8	0.8	1.36	1.70
49.	236	0.30	0.30	0.30	0.10	0	0	C ₂₃₆	0.62	1	0.8	0.8	1.45	1.83
50.	245	0.30	0.30	0.30	0	0.10	0	C ₂₄₅	0.63	1	0.8	0.8	1.48	1.77
51.	246	0.30	0.30	0	0.30	0.10	0	C ₂₄₆	0.67	1	0.8	0.8	1.42	1.80
52.	256	0.30	0	0.30	0.30	0.10	0	C ₂₅₆	0.65	1	0.8	0.8	1.30	1.77
53.	345	0	0.30	0.30	0.30	0.10	0	C ₃₄₅	0.63	1	0.8	0.8	1.27	1.71
54.	346	0.10	0.30	0.30	0.30	0	0	C ₃₄₆	0.62	1	0.8	0.8	1.31	1.79
55.	356	0.30	0.10	0.30	0.30	0	0	C ₃₅₆	0.64	1	0.8	0.8	1.33	1.83
56.	456	0.30	0.10	0.30	0.30	0	0	C ₄₅₆	0.65	1	0.8	0.8	1.41	1.85

The actual component as transformed from Eqn. (43), Table (1) and (2) were used to measure out the quantities of Water/Cement ratio (Z_1), Cement (Z_2), CPA(Z_3), RHA(Z_4), Fine Aggregate (Z_5) and Coarse Aggregate (Z_6) in their respective ratios for the concrete cube strength test.

3. MATERIALS and METHODS

3.1. MATERIALS

In this research work, the constituent materials under investigation in line with Scheffe's six component mixture, degree three are Water/Cement ratio, Cement, CPA, RHA, Fine and Coarse Aggregates. The water is obtained from potable water from the clean water source. The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). As in the case of previous work on CPA-RHA – Cement Concrete based on Scheffe's second degree model by Nwachukwu and others (2022i), only 60% of the cement is being replaced by CPA and RHA. So the quantity of (C:CPA:RHA) by weight is measured out in the ratio of (0.4:0.3:0.3). The same procedure involved in obtaining and preparing CPA and RHA in the previous CPA-RHA- Cement Concrete work by Nwachukwu and others (2022i) was also applied in this present Scheffe's (6,3) based work. Fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent

leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 112 mix ratios were to be used to produce 224 prototype concrete cubes. Fifty- six (56) out of the 112 mix ratios were used as control mix ratios to produce 112 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (38). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. Two samples were crushed for each mix ratio in this present study. In each case, the compressive strength was then calculated using Eqn.(45)

$$\text{Compressive Strength} = \frac{\text{Average failure Load, } P \text{ (N)}}{\text{Cross- sectional Area, } A \text{ (mm}^2\text{)}} \quad (45)$$

4. RESULTS PRESENTATION and DISCUSSION

4.1. PRESENTATION of COMPRESSIVE STRENGTH RESULTS of CPA-RHA-CEMENT CONCRETE BASED ON SCHEFFE'S (6,3) MODEL FOR THE INITIAL EXPERIMENTAL TESTS.

The results of the compressive strength (R_{response}, N_i) based on a 28-days strength is presented in Table 3. These are calculated from Eqn.(45)

Table 3: 28th Day Compressive Strength Test Results of CPA-RHA-Cement Concrete Based on Scheffe's (6,3) Model for the Initial Experimental Tests.

S/N	POINTS	EXPERIMENTAL NUMBER	RESPONSE N_i , MPa	RESPONSE SYMBOL	ΣN_i	AVERAGE RESPONSE N , MPa
1.	1	1C	40.21	N_1	81.25	40.63
		1D	41.04			
2.	2	2C	34.89	N_2	69.65	34.83
		2D	34.76			
3.	3	3C	40.00	N_3	79.86	39.93
		3D	39.86			
4.	4	4C	37.88	N_4	75.90	37.95
		4D	38.02			
5.	5	5C	29.08	N_5	59.12	29.56
		5D	30.04			
6.	6	6C	36.55	N_6	73.19	36.60
		6D	36.64			
7.	112	7C	41.04	N_{112}	85.12	41.06
		7D	41.08			
8.	122	8C	39.65	N_{122}	78.61	39.31
		8D	38.96			
9.	113	9C	31.85	N_{113}	63.68	31.84
		9D	31.83			
10.	133	10C	42.12	N_{133}	83.98	41.99
		10D	41.86			
11.	114	11C	38.05	N_{114}	76.13	38.07
		11D	38.08			
12.	144	12C	34.32	N_{144}	69.42	34.71
		12D	35.10			
13.	115	13C	32.34	N_{115}	65.42	32.71
		13D	33.08			

14.	155	14C	36.76	N ₁₅₅	73.54	36.77
		14D	36.78			
15.	116	15C	35.53	N ₁₁₆	71.62	35.81
		15D	36.09			
16.	166	16C	41.53	N ₁₆₆	82.99	41.50
		16D	41.46			
17.	223	17C	38.54	N ₂₂₃	77.62	38.81
		17D	39.08			
18.	233	18C	33.66	N ₂₃₃	67.30	38.65
		18D	33.64			
19.	224	19C	29.86	N ₂₂₄	60.00	30.00
		19D	30.14			
20.	244	20C	34.42	N ₂₄₄	69.63	34.82
		20D	35.21			
21.	225	21C	42.05	N ₂₂₅	83.63	41.82
		21D	41.58			
22.	255	22C	29.08	N ₂₅₅	59.14	29.57
		22D	30.06			
23.	226	23C	31.46	N ₂₂₆	62.94	31.47
		23D	31.48			
24.	266	24C	32.64	N ₂₆₆	65.70	32.85
		24D	33.06			
25.	334	25C	36.08	N ₃₃₄	72.84	36.42
		25D	36.76			
26.	344	26C	35.43	N ₃₄₄	70.88	35.44
		26D	35.45			
27.	335	27C	37.43	N ₃₃₅	74.97	37.49
		27D	37.54			
28.	355	28C	38.85	N ₃₅₅	77.74	38.87
		28D	38.89			
29.	336	29C	36.43	N ₃₃₆	72.88	36.44
		29D	36.45			
30.	366	30C	39.04	N ₃₆₆	79.05	39.53
		30D	40.01			
31.	445	31C	28.98	N ₄₄₅	57.86	28.93
		31D	28.88			
32.	455	32C	29.56	N ₄₅₅	60.21	30.11
		32D	30.65			
33.	446	33C	31.08	N ₄₄₆	62.17	31.09
		33D	31.09			
34.	466	34C	34.23	N ₄₆₆	69.35	34.68
		34D	35.12			
35.	556	35C	38.87	N ₅₅₆	77.41	38.71
		35D	38.54			
36.	566	36C	35.34	N ₅₆₆	71.46	35.73
		36D	36.12			
37.	123	37C	29.54	N ₁₂₃	59.40	29.70
		37D	29.86			
38.	124	38C	31.32	N ₁₂₄	62.66	31.33
		38D	31.34			
39.	125	39C	38.32	N ₁₂₅	77.43	38.72
		39D	39.11			

40.	126	40C	40.21	N ₁₂₆	80.53	40.27
		40D	40.32			
41.	134	41C	42.32	N ₁₃₄	85.38	42.69
		41D	43.06			
42.	135	42C	32.21	N ₁₃₅	64.55	32.28
		42D	32.34			
43.	136	43C	43.77	N ₁₃₆	87.50	43.75
		43D	43.73			
44.	145	44C	37.43	N ₁₄₅	75.47	37.74
		44D	38.04			
45.	146	45C	38.08	N ₁₄₆	76.16	38.08
		45D	38.32			
46.	156	46C	41.08	N ₁₅₆	81.40	40.70
		46D	40.32			
47.	234	47C	34.86	N ₂₃₄	70.07	35.04
		47D	35.21			
48.	235	48C	35.32	N ₂₃₅	70.73	35.37
		48D	35.41			
49.	236	49C	37.23	N ₂₃₆	74.55	37.28
		49D	37.32			
50.	245	50C	38.44	N ₂₄₅	76.98	38.49
		50D	38.54			
51.	246	51C	36.23	N ₂₄₆	73.44	36.72
		51D	37.21			
52.	256	52C	38.32	N ₂₅₆	77.44	38.72
		52D	39.12			
53.	345	53C	38.12	N ₃₄₅	76.33	38.17
		53D	39.21			
54.	346	54C	28.46	N ₃₄₆	56.88	28.44
		54D	28.42			
55.	356	55C	38.46	N ₃₅₆	76.95	38.48
		55D	38.49			
56.	456	56C	41.06	N ₄₅₆	83.08	41.54
		56D	42.02			

4.2. PRESENTATION of COMPRESSIVE STRENGTH RESULTS of CPA-RHA-CEMENT CONCRETE BASED ON SCHEFFE'S (6,3) MODEL FOR THE EXPERIMENTAL (CONTROL) TEST.

Table 4 shows the 28th day Compressive strength results for the Experimental (Control) Test

Table 4: 28TH Day Compressive Strength Results Based on Scheffe's (6,3) Model for the Experimental (Control) Tests.

S/N	CONTROL POINTS	EXPERIMENTAL NUMBER	RESPONSE, MPa	AVERAGE RESPONSE, MPa
1.	C ₁	1C	38.98	39.03
		1D	39.08	
2.	C ₂	2C	35.77	35.72
		2D	35.66	
3.	C ₃	3C	41.22	41.27
		3D	41.31	
4.	C ₄	4C	36.55	36.82
		4D	37.08	
5.	C ₅	5C	31.22	31.27
		5D	31.32	

6.	C_6	6C	38.33	38.38
		6D	38.43	
7.	C_{112}	7C	43.21	43.10
		7D	42.98	
8.	C_{122}	8C	38.32	38.30
		8D	38.28	
9.	C_{113}	9C	32.22	32.28
		9D	32.33	
10.	C_{133}	10C	41.22	41.28
		10D	41.34	
11.	C_{114}	11C	36.33	36.34
		11D	36.34	
12.	C_{144}	12C	34.88	34.73
		12D	34.58	
13.	C_{115}	13C	30.54	30.43
		13D	30.32	
14.	C_{155}	14C	37.86	37.87
		14D	37.87	
15.	C_{116}	15C	35.45	35.44
		15D	35.43	
16.	C_{166}	16C	39.47	39.48
		16D	39.48	
17.	C_{223}	17C	37.65	37.76
		17D	37.86	
18.	C_{233}	18C	32.22	32.33
		18D	32.43	
19.	C_{224}	19C	28.23	28.67
		19D	29.11	
20.	C_{244}	20C	35.66	35.76
		20D	35.86	
21.	C_{225}	21C	41.12	41.15
		21D	41.18	
22.	C_{255}	22C	30.21	30.22
		22D	30.22	
23.	C_{226}	23C	31.42	31.45
		23D	31.48	
24.	C_{266}	24C	30.67	30.73
		24D	30.78	
25.	C_{334}	25C	34.54	34.59
		25D	34.64	
26.	C_{344}	26C	32.23	32.68
		26D	33.12	
27.	C_{335}	27C	37.25	37.18
		27D	37.11	
28.	C_{355}	28C	39.21	39.16
		29C	39.11	
29.	C_{336}	29D	35.32	35.33
		29D	35.33	
30.	C_{366}	30C	40.22	40.23
		30D	40.24	
31.	C_{445}	31C	29.34	29.79
		31D	30.23	

32.	C ₄₅₅	32C	31.54	31.59
		32D	31.64	
33.	C ₄₄₆	33C	30.08	30.10
		33D	30.12	
34.	C ₄₆₆	34C	32.12	32.57
		34D	33.02	
35.	C ₅₅₆	35C	42.12	41.98
		35D	41.84	
36.	C ₅₆₆	36C	34.34	34.40
		36D	34.45	
37.	C ₁₂₃	37C	28.32	28.33
		37D	28.34	
38.	C ₁₂₄	38C	30.23	30.28
		38D	30.32	
39.	C ₁₂₅	39C	37.34	37.34
		39D	37.38	
40.	C ₁₂₆	40C	39.54	39.56
		40D	39.58	
41.	C ₁₃₄	41C	40.32	40.78
		41D	41.23	
42.	C ₁₃₅	42C	30.43	30.44
		42D	30.45	
43.	C ₁₃₆	43C	41.23	40.68
		43D	40.12	
44.	C ₁₄₅	44C	35.34	35.41
		44D	35.47	
45.	C ₁₄₆	45C	39.33	38.91
		45D	38.48	
46.	C ₁₅₆	46C	39.08	39.54
		46D	40.00	
47.	C ₂₃₄	47C	34.86	34.93
		47D	35.00	
48.	C ₂₃₅	48C	36.86	36.71
		48D	36.56	
49.	C ₂₃₆	49C	38.43	38.41
		49D	38.38	
50.	C ₂₄₅	50C	36.98	36.86
		50D	36.74	
51.	C ₂₄₆	51C	35.45	35.74
		51D	36.02	
52.	C ₂₅₆	52C	36.45	36.34
		52D	36.23	
53.	C ₃₄₅	53C	40.23	40.36
		53D	40.48	
54.	C ₃₄₆	54C	29.44	29.38
		54D	29.32	
55.	C ₃₅₆	55C	39.32	39.33
		55D	39.34	
56.	C ₄₅₆	56C	41.32	41.35
		56D	41.38	

4.3. SCHEFFE'S (6,3) REGRESSION MODEL FOR THE CPA-RHA-CEMENT CONCRETE RESPONSES

Substituting the values of the compressive strengths (responses) from Table 3 into Eqns.(8) through (38), we obtain the coefficients ($\beta_1, \beta_2, \beta_3, \dots, \beta_{12}, \beta_{13}, \dots, \beta_{123}, \beta_{124}, \dots, \beta_{456}, \dots, \gamma_{12}, \gamma_{13}, \gamma_{56}$), in MPa of the Scheffe's third degree polynomial. Substituting the values of these above coefficients into Eqn. (7), we obtain the regression model for the optimization of the compressive strength of the CPA-RHA-Cement Concrete based on Scheffe's (6,3) lattice.

4.4. SCHEFFE'S (6,3) MODEL RESPONSES FOR CPA-RHA-CEMENT CONCRETE AT CONTROL POINTS

By substituting the pseudo mix ratio of points $C_1, C_2, C_3, C_4, \dots, C_{112}, \dots, C_{456}$ of Table 2 into revised Eqn.(7), we obtain the third degree model responses for the control points for the CPA-RHA-Cement Concrete.

4.5. TEST of ADEQUACY of THE SCHEFFE'S (6,3) MODEL FOR THE CPA-RHA-CEMENT CONCRETE

Here, the Student's - T - test is used order to check if there is any significant difference between the compressive strength results (lab responses) given in Table 4 and model responses from the control points evaluated through session 4.4. The procedures for using the Student's - T - test have been explained by Nwachukwu and others (2022 c). The result of the test confirms the adequacy of the model. Therefore, the Scheffe's model is validated and is very adequate for predicting the compressive strength of CPA-RHA-Cement Concrete based on Scheffe's (6,3) simplex lattice.

4.6. RESULTS DISCUSSION

The Optimum attainable compressive strength of the CPA-RHA Concrete Mixture based on Scheffe's (6,3) lattice is 43.75MPa at N_{136} . This corresponds to mix ratio of 1.8:1.0:0.8:0.8:0.4:5.0 for Water/Cement Ratio, Cement, CPA, RSA, Fine Aggregate and Coarse Aggregate respectively. Similarly, the lowest compressive strength was found to be 28.44MPa at N_{346} , which also correspond to the mix ratio of 2.2:1.0:0.8:0.8:3.2:4.8 for Water/Cement Ratio, Cement, CPA, RSA, Fine Aggregate and coarse Aggregate respectively. The maximum value from the model was found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete and also minimum standard (of 4500psi or 30.75MPa) specified by the American

Society of Testing and Machine, ASTM C 39 and ASTM C 469. Subsequently, using the model, all compressive strength of all points (1 - 56) in the simplex can be evaluated based on Scheffe's Third Degree Model.

5. CONCLUSION

Scheffe's Third Degree Regression Model, for six component, degree three mixtures, Scheffe's (6,3) has been presented so far and used to predict the mix proportions as well as a model for predicting the compressive strength of CPA-RHA-Cement Concrete cubes. By using Scheffe's (6,3) simplex model, the values of the compressive strength were obtained for the present concrete mixture at all 56 points. The result of the student's t-test confirmed that there is a good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The maximum compressive strength predicted by the Scheffe's (6,3) model is 43.75 MPa which is slightly higher the maximum value of 39.77MPa obtained by Nwachukwu and others (2022i) based on the Scheffe's second degree model. The minimum value obtained from the present work is 28.44 MPa. However, both maximum values meet the minimum standard requirement (of 20 MPa and 30.75MPa) stipulated by American Concrete Institute (ACI) and American Society of Testing and Machine, ASTM C 469 or ASTM C 39 respectively, for the compressive strength of good concrete. Thus, with the Scheffe's (6,3) model, any desired strength of the present concrete mixture given any mix proportions can be easily predicted and evaluated and vice versa. By the utilization of this Scheffe's optimization model, the problem of having to go through vigorous, time-consuming and laborious mixture design procedures to obtain the desired strength has been reduced to the barest minimum. Finally, it can be deduced that the RHA and CPA compositions that should have posed as a menace to the environment is now being utilized as a substantial cement part replacements with capacities of increasing the concrete's compressive strength.

REFERENCES

- [1] ACI Committee 211. (1994): "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211. 1R-91)"; American Concrete Institute, ACI Manual of Concrete Practice, Detroit, Michigan, USA.
- [2] ACI Committee 544. (1982): "State-of-the-Report on Fibre Reinforced Concrete, (ACI 544. 1R-82)"; *Concrete International: Design*

- and Construction. Vol. 4, No. 5: Pp. 9-30, American Concrete Institute, Detroit, Michigan, USA.
- [3] ACI Committee 544. (1989): "Measurement of Properties of Fibre Reinforced Concrete, (ACI 544. 2R-89)"; American Concrete Institute, Detroit, Michigan, USA.
- [4] Adetoye, O. A., Afolayan, T. J. and Asekunowo, T. (2022): "Compressive Strength Properties of Cassava Peel Ash and Wood Ash in concrete Production, "; International Journal of New Practices in Management and Engineering, Vol. 11, No. 1.
- [5] Aggarwal, M. L. (2002): "Mixture Experiments: Design Workshop Lecture Notes", University of Delhi, India.
- [6] ASTM C 39 (1997), Standard Test Method for Compressive Strength of Concrete Cubes, American Society for Testing and Material, West Conshohocken, PA, USA
- [7] ASTM C469 (2010), Standard Test Method for Compressive Strength of Concrete Cubes, American Society for Testing and Material, West Conshohocken, PA, USA
- [8] British Standards Institution, BS 12 (1978): Ordinary and Rapid – Hardening Portland cement; London.
- [9] British Standards Institution, BS 1881-Part 116 (1983). Methods of Determination of Compressive Strength of Concrete Cube, London.
- [10] Department of Environment, DOE (1988): Design of Normal Concrete Mixes, HMSO, London.
- [11] Ezech, J. C. & Ibearugbulam, O. M. (2009): "Application of Scheffe's Model in Optimization of Compressive Cube Strength of River Stone Aggregate Concrete"; *International Journal of Natural and Applied Sciences*; Vol. 5, No. 4, Pp 303 – 308 .
- [12] Ezech, J. C., Ibearugbulam, O. M. & Anyaogu, L. (2010a): "Optimization of Compressive Strength of Cement- Sawdust Ash Sandcrete Block using Scheffe's Mathematical Model"; *Journal of Applied Engineering Research*. Vol. 4, No. 4, Pp 487–494.
- [13] Ezech, J. C., Ibearugbulam, O. M. & Anya, U. C (2010b): "Optimization of aggregate composition of laterite/sand hollow Block Using Scheffe's Simplex Method"; *International Journal of Engineering*. Vol. 4, No. 4, Pp 471 – 478.
- [14] Hughes, B. P. (1971): The Economic Utilization of Concrete Materials, Proceedings of the Symposium on Advances in Concrete, Concrete Society, London
- [15] Ibearugbulam, O. M. (2006): "Mathematical Model for Optimization of Compressive Strength of Periwinkle Shell-Granite Aggregate Concrete"; *M. Eng. Thesis*, Federal University of Technology, Owerri, Nigeria.
- [16] Jackson, N. and Dhira, R. K. (1996): Civil Engineering Materials, 5th Edition, Palgrave, New York.
- [17] Mohd- Asharuddin, S, Othman, N., Mohd Zin, N. S. and Tajarudin, H. A. (2017): "Chemical and Morphological Study of Cassava Peel: A Potential Waste as Coagulant Aid; MATEC Web of
- [18] Neville, A. M. (1990): Properties of Concrete; 3rd edition, Pitman, England.
- [19] Nwachukwu, K. C., Njoku, K. O., Okorie, P. O., Akosubo, I. S., Uzoukwu, C. S., Ihemegbulam, E. O. and Igbojiaku, A. U (2022a): "Comparison Between Scheffe's Second Degree (5, 2) and Third Degree (5, 3) Polynomial Models In The Optimization of Compressive Strength of Glass Fibre Reinforced Concrete (GFRP)"; American Journal of Computing and Engineering (AJCE), Vol. 5, No. 1, Pp 1– 23.
- [20] Nwachukwu, K. C., Okafor, M., Thomas, B., Oputa, A. A., Okodugha, D. A. and Osaigbovo, M. E. (2017): "An Improved Model For The Optimization of The Compressive Strength of Glass Fibre Reinforced Concrete (GFRP) Using Scheffe's Second Degree Polynomials"; Research journal's Journal of Civil Engineering, Vol. 3, No. 2
- [21] Nwachukwu, K. C., Okodugha, D. A., Uzoukwu, C. S., Igbojiaku, A. U. and Okafor, M. (2022b): "Application of Scheffe's Second Degree Mathematical Model For The Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC)"; International Journal of Advances in Engineering and Management (IJAEM), Vol. 4, No. 3.
- [22] Nwachukwu, K. C., Okorie, P. O. Ihemegbulam, E. O., Kemebiye Dimaro,

- Uzoukwu, C. S., and Maduagwu, J. C. (2022c): "The Use of Scheffe's Model For The Optimization of Compressive Strength of Polypropylene Fibre Reinforced Concrete (PFRC)"; *International Journal of Innovative Science and Research Technology (IJISRT)*, Vol. 7, No. 1
- [23] Nwachukwu, K. C., Ozioko, H. O., Okorie, P. O., and Uzoukwu, C. S. (2022d): "Application of Scheffe's Mathematical Model For The Optimization of Compressive Strength of Nylon Fibre Reinforced Concrete (NFRC)"; *International Journal of Advances in Engineering and Management (IJAEM)*, Vol. 4, No. 2, Pp 850-862
- [24] Nwachukwu, K. C., Okodugha, D. A., Igbojiaku, A. U., Attah, U. G. and Ihemegbulem, E. O. (2022e): Application of Scheffe's Third Degree Regression Model For The Optimization of Compressive Strength of Polypropylene Fibre Reinforced Concrete (PFRC); *International Journal of Advance Research and Innovative Ideas In Education (IJARIIE)*, Vol. 8, No. 2.
- [25] Nwachukwu, K. C., Okodugha, D. A., Edike, O., Okafor, M. and Attah, U. G. (2022f): Application of Modified Scheffe's Third Degree Polynomial Model For The Optimization of Compressive Strength of Nylon Fibre Reinforced Concrete (NFRC); *International Journal of Trend In Scientific Research and Development (IJTSRD)*, Vol. 6, No. 3.
- [26] Nwachukwu, K. C., Okodugha, D. A., Akosubo, I. S. and Atulomah, F. K (2022g): Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC) Using Scheffe's Third-Degree Regression Model; *Iconic Research and Engineering Journals (IRE)*, Vol. 5, No. 11
- [27] Nwachukwu, K. C., Onwuegbuchulem, U. C., Atulomah" F. K. and Ihemegbulem, E. O. (2022h): Optimization of Hybrid-Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC); *Iconic Research and Engineering Journals (IRE)*, Vol. 5, No. 12
- [28] Nwachukwu, K. C., Thomas' B., Oguaghamba, O., Onwuegbuchulem, U. C., Igbojiaku, A. U., Atulomah, F. K and Okafor. M. (2022i): Optimization of Compressive Strength of Concrete Made With Partial Replacement of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's Second Degree Model; *international Journal of Engineering Inventions (IJEI)*, Vol. 11, No. 8, Pp. 40-50.
- [29] Nwakonobi, T. U and Osadebe, N. N (2008): "Optimization Model for Mix Proportioning of Clay- Ricehusk- Cement Mixture for Animal Buildings"; *Agricultural Engineering International: the CIGRE journal, Manuscript BC 08 007*, Vol x
- [30] Obam, S. O. (2006). The Accuracy of Scheffe's Third Degree over Second Degree Optimization Regression Polynomials, *Nigerian Journal of Technology*, Vol. 2, No. 25, Pp 1 – 10.
- [31] Obam, S. O. (2009): "A Mathematical model for Optimization of Strength of concrete: A case study for shear modulus of Rice Husk Ash Concrete." *Journal of Industrial Engineering International I*; Vol. 5, No. 9, Pp 76 – 84.
- [32] Obute, J. O., Akin, O. O., Amartey, V. D. and Ejeh, S. P (2021): "Effect of the Partial Replacement of Cement with Cassava Peel Ash and Rice Husk Ash on Concrete "; *Covenant Journal of Engineering Technology(CJET)*, Vol. 5, No. 1, Pp 20 – 28.
- [33] Ogbonna, C, Mbadike, E and Alaneme, G. U. (2020): "Characteristics and Use of Cassava Peel Ash In Concrete Production"; *CEPM Journal of Computational Engineering and Physical Modelling*, Vol. 3, No. 2, Pp 12 – 28.
- [34] Okere, C. E., (2006): "Mathematical Models for Optimization of the Modulus of rupture of concrete"; M. Eng. Thesis, Civil Engineering Department, Federal University of Technology, Owerri.
- [35] Olaokunbo, O., Anthony, E., Rotimi, O., Solomon, O., Tolulope, A, and John, O. (2018): "Assessment of Strength Properties of Cassava Peel Ash - Concrete"; *International Journal of Civil Engineering and Technology (IJCET)*, Vol. 9, No. 1, Pp 965 – 974.
- [36] Onwuka, D. O, Okere, C. E., Arimanwa, J. I. and Onwuka, S. U. (2011): "Prediction of Concrete Mix ratios using Modified Regression Theory, *Computer Methods in Civil Engineering*, Vol. 2. No. 1 Pp. 95- 107.
- [37] Oyenuga, V. O. (2008): *Simplified Reinforced Concrete Design*, 2nd Edition, Asros Ltd, Lagos, Nigeria.

- [38] Patel, P. A., Desai, A. K. & Desai, J. A (2012): "Evaluation of Engineering Properties for Polypropylene Fibre Reinforced Concrete"; International Journal of Advanced Engineering Technology (IJAET) Vol. 3, No. 1, Pp 42 – 45.
- [39] Raheem, S. B., Arubike, E. D. and Awogboroi, O. S. (2015): "Effects of Cassava Peel Ash(CPA) as alternative Binder in Concrete"; International Journal of Constructive Research In Civil Engineering (IJCRCE), Vol. 1, No. 2, Pp 27 – 32.
- [40] Scheffe, H. (1958): Experiment with Mixtures"; *International Journal of Royal Statistics Society*, Series B, Vol. 20, Pp. 344-360.
- [41] Shacklock, B. W. (1974): Concrete Constituents and Mix Proportions, the Cement and Concrete Association, London.
- [42] Shetty, M. S. (2006): Concrete Technology (Theory and Practice), S. Chand and Company Ltd, India.
- [43] Syal, I. C. and Goel, A. K. (2007): Reinforced Concrete Structures, S. Chand, India.
- [44] Zareei, S. A., Ameri, F., Dorostkar, F. and Ahmadi, M (2017): "Rice Husk Ash As A Partial Replacement of cement In High Strength concrete containing micro silica: Evaluating Durability and Mechanical properties; Science Direct – case studies in construction materials, Vol. 7, Pp 73 – 81.

